



3U MOCK EXAM

2020

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General

Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks:

70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–14)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 Marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Q1-10

1 What is the derivative of $\sin^{-1}\frac{x}{2}$?

A. $\frac{1}{\sqrt{4-x^2}}$

B. $-\frac{1}{\sqrt{4-x^2}}$

C. $\frac{4}{\sqrt{4-x^2}}$

D. $-\frac{4}{\sqrt{4-x^2}}$

2 What is the remainder when $2x^3 - 9x^2 + 3x + 73$ is divided by $x + 5$?

A. -419

B. $-x^2 + 9x + 12$

C. -417

D. $2x^2 - 19x + 98$

3 Find the domain of the function $f(x) = \ln(6 - 2x)$

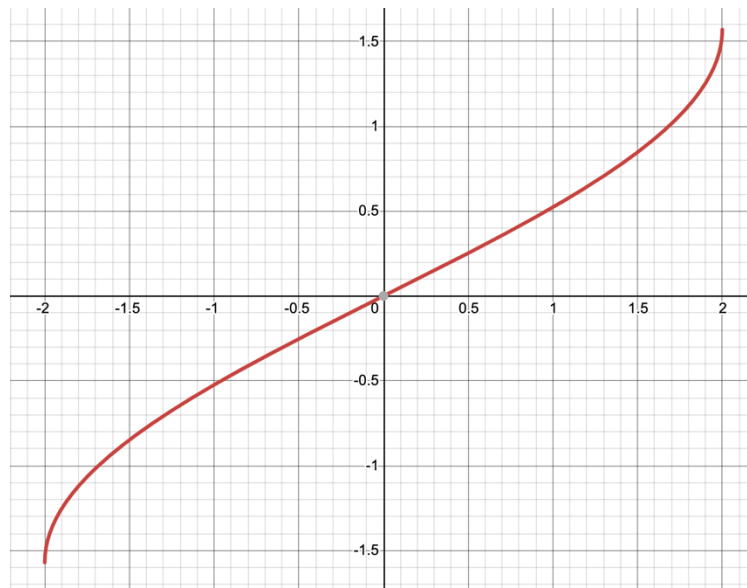
A. $x < 5$

B. $x > 6$

C. $x < 3$

D. $x > \frac{1}{3}$

4 The diagram shows the graph of $y = f(x)$



Which equation best describes the graph?

- A. $y = 2 \sin^{-1} x$
- B. $y = \sin^{-1} \frac{x}{2}$
- C. $y = \frac{1}{2} \sin^{-1} x$
- D. $y = \sin^{-1} 2x$

5 Given $\vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$, which of the following best describes unit vector \hat{a} ?

- A. $\frac{3}{2}\vec{i} + \frac{5}{2}\vec{j}$
- B. $3\vec{i} + 4\vec{j}$
- C. $\frac{1}{3}\vec{i} + \frac{1}{4}\vec{j}$
- D. $\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

6 For a discrete distribution $X \sim (n, p)$, which of the following best describes the 'Expected Value'?

- A. $E(X) = p$
- B. $E(X) = np$
- C. $E(X) = np(1 - p)$
- D. $E(X) = \sqrt{np(1 - p)}$

7 Consider the separable differential equation $\frac{dx}{dy} = x^5 y^{21}$

To solve for a general solution, the differential equation is rewritten into the form

$$f(y)dy = g(x)dx$$

Which of the following best describes the function $g(x)$?

- A. x^5
- B. x^{-5}
- C. x^{-5}
- D. y^{21}

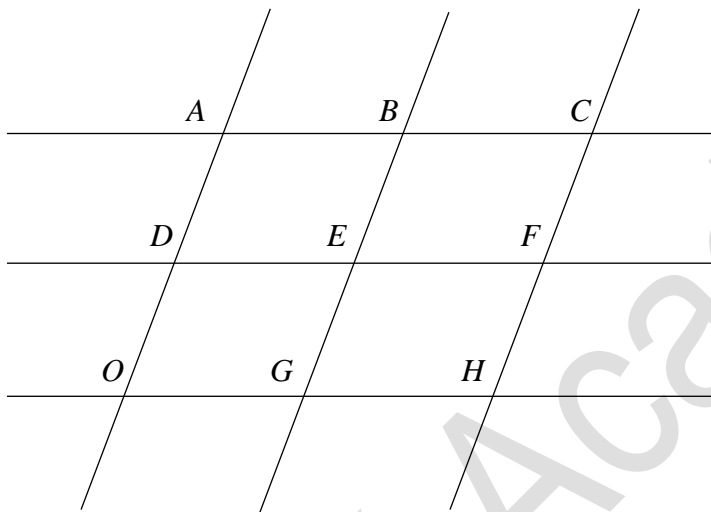
8 In how many ways can all the letters of the word PARALLEL be placed in a line with the three Ls together?

- A. $\frac{6!}{2!}$
- B. $\frac{6!}{2!3!}$
- C. $\frac{8!}{2!}$
- D. $\frac{8!}{2!3!}$

9 What is the coefficient of x^4 in the expansion of $\left(3x + \frac{2}{x}\right)^6$?

- A. 2196
- B. 3264
- C. 3462
- D. 2916

10 Consider the following diagram



Given that ABC, DEF, and OGH are equally spaced parallel lines, as are ADO, BEG, and CFH, and P is the midpoint of AD,

If $\overrightarrow{OH} = \vec{h}$ and $\overrightarrow{OA} = \vec{a}$, which of the following expressions represents vector \overrightarrow{GP} ?

- A. $\frac{1}{2}\vec{a} - \frac{3}{4}\vec{h}$
- B. $\frac{1}{2}\vec{h} - \frac{3}{4}\vec{a}$
- C. $\frac{3}{4}\vec{a} - \frac{1}{2}\vec{h}$
- D. $\frac{3}{4}\vec{h} - \frac{1}{2}\vec{a}$

End of Section I

Section II

60 Marks

Attempt Questions 11-14

Allow about 1 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

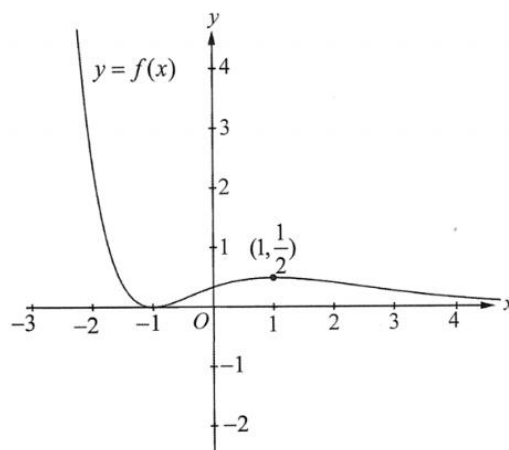
In Questions 11-14, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

- (a) Find the acute angle between the following vectors:
- (i) $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
 $v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ 1
- (ii) $u = 9\hat{i} + 3\hat{j}$
 $v = 3\hat{i} + 3\hat{j}$ 1
- (b) Find $\int 3 \cos^2 3x \, dx$ 2
- (c) Solve for x , $\frac{1}{x+1} < 3$ 2
- (d) A survey was conducted at PEAK Academy to decide whether students preferred Mathematics or English. 34% of students were in favour of English, while the remaining 66% were in favour of Mathematics. Assume the survey was conducted on a large sample size.
If 5 survey responses were chosen at random, find the probability that the majority of the responses were in favour of Mathematics. 2
- (e) Given that $P(x) = ax^3 + bx^2 - 19x - 15$ evaluate a and b if $(x + 1)$ and $(x - 3)$ are both factors of $P(x)$ 3

Question 11 continues on the next page

(f) The diagram shows the graph of $y = f(x)$



Draw separate one-third page diagrams for each of the following functions.

- (i) $y = \frac{1}{f(x)}$ 2
- (ii) $y = \sqrt{f(x)}$ 2

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet

- (a) Prove by induction that $2^n + 1$ is divisible by 3 for all odd integers. 3
- (b) A box has 7 green marbles and 3 black marbles. A marble is randomly selected from the box, its colour noted, and then put back in the box. If X is the random variable that represents the number of black marbles selected in 12 trials, calculate $P(X \leq 4)$ 3
- (c) A yacht sails from a pier in the direction of $N25^\circ E$ for 15km, then turns to $N58^\circ W$ for 12km and finally $S20^\circ W$ for 28km. Using your knowledge on Vectors, what distance and bearing is the yacht from the pier? 3
- (d) Find the general solutions, or particular solution if any, of the following:
- (i) $\frac{dy}{dx} = e^{x+y}$ 1
- (ii) $\frac{dy}{dx} = x^2(1 + y^2)$ 1
- (iii) $\frac{dy}{dx} = 3x^2y^2$ $y = 1, x = 0$ 1
- (e) By considering the expansion of $x(1 + x)^n$, prove

$$\sum_{r=0}^n (r + 1) \binom{n}{r} = 2^n \left(\frac{n}{2} + 1 \right)$$

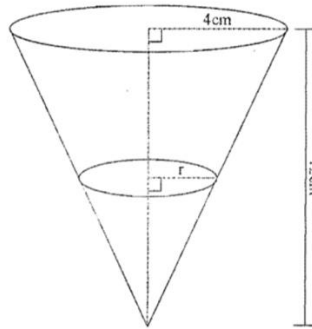
3

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet

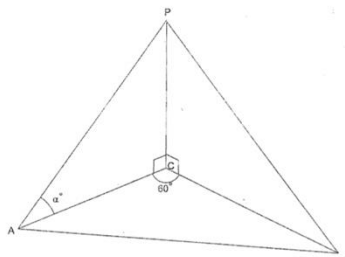
- (a) Solve $\sec^2 x + \tan x - 7 = 0$ for $0^\circ \leq x \leq 360^\circ$. Give your answers correct to the nearest minute. **3**

- (b) The diagram shows a conical drinking cup of height 12cm and radius 4cm. The cup is filled with water at a rate of 3cm^3 per second. The height of water at time t seconds is h cm and the radius of the water's surface is r cm.



- (i) Show that $r = \frac{1}{3}h$ **1**
- (ii) Find the rate at which the height is increasing when the height of the water is 9cm. ($V = \frac{1}{3}\pi r^2 h$ is the volume of a cone) **3**

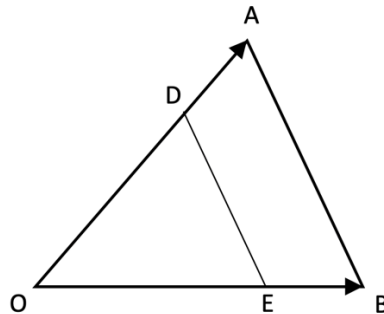
- (c) The position of two boats, A and B out at sea subtend an angle of 60° at the base C of a cliff. The distance AC is 3 times the height of the cliff and the distance BC is 4 times the height of the cliff.



- (i) Show that the angle of elevation α of the cliff from point A is $18^\circ 26'$ **1**
- (ii) The distance AB is 300 metres greater than the height of the cliff. Find the height of the cliff. **3**

Question 13 continues on the next page

- (d) Let D, E be points dividing the sides OA and OB of a triangle OAB in the ratio $\lambda:1-\lambda$, where $0 < \lambda < 1$. Prove that DE is parallel to AB and the length of DE is λ times that of AB .



2

- (e) Using all the letters of MATHEMATICS, how many different arrangements can be made?

2

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

- (a) Prove by induction that it is possible to cover a $2^n \times 2^n$ grid with L tiles, consisting of 3 squares, if 1 tile is removed. **4**
- (b) A dead body is discovered by a cleaner in a hotel room at 11am. A forensic scientist arrives at the scene at 11:30am and records the temperature of the body at 24.5°C . The body temperature is recorded again an hour later at 24°C . Estimate the time of death given that the hotel room is at a constant temperature of 20°C and normal body temperature is 36.5°C . **4**
- (c) A certain type of glass is such that a slab 1cm thick absorbs a quarter of the light passing through it. How thin must the pane be such that only 1 percent of light is absorbed? **3**
- (d) In a random group of 25 people, what is the probability that at least two of them have the same birthday? You may assume that there is an equal likelihood of being born on any given day in a 365 day year. **4**

End of paper